AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

I declare this is my own work.

# AS MATHEMATICS

Paper 2

Time allowed: 1 hour 30 minutes

**Materials**
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

**Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

**Information**
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

| For Examiner's Use | |
|---|---|
| Question | Mark |
| 1 | |
| 2 | |
| 3 | |
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| 17 | |
| 18 | |
| **TOTAL** | |

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## Section A

Answer **all** questions in the spaces provided.

**1** Express as a single power of $a$

$$\frac{a^2}{\sqrt{a}}$$

where $a \neq 0$

Circle your answer.

**[1 mark]**

$a^1$ $a^{\frac{3}{2}}$ $a^{\frac{5}{2}}$ $a^4$

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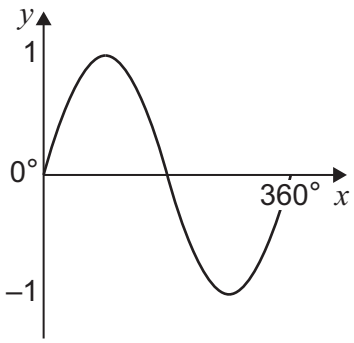
**2** One of the diagrams below shows the graph of $y = \sin(x + 90°)$ for $0° \leq x \leq 360°$

Identify the correct graph.

Tick (✓) **one** box.

**[1 mark]**

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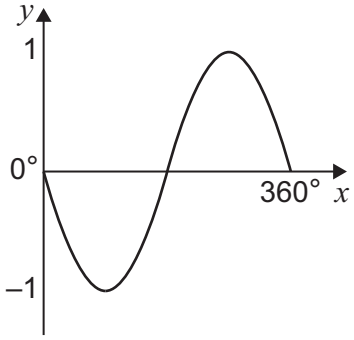


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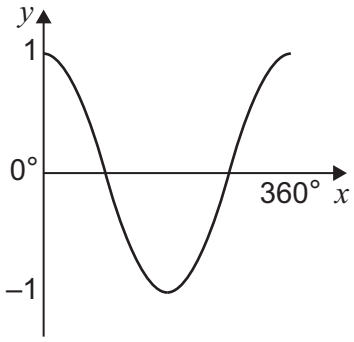


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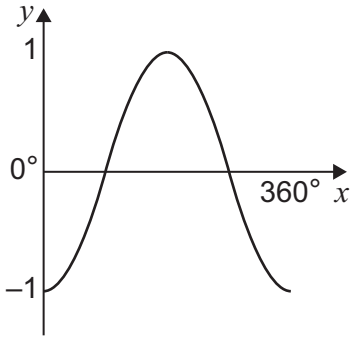


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**3** It is given that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \sqrt{x}$$

Find an expression for $y$.

**[3 marks]**

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**4 (a)** Find the binomial expansion of $(1 - 2x)^5$ in ascending powers of $x$ up to and including the term in $x^2$

**[2 marks]**

**4 (b)** Find the first two non-zero terms in the expansion of

$$(1 - 2x)^5 + (1 + 5x)^2$$

in ascending powers of $x$.

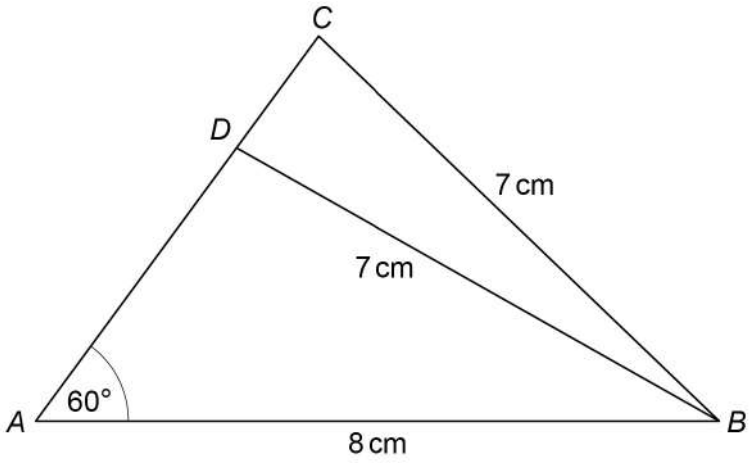
**[2 marks]**

**4 (c)** Hence, use an appropriate value of $x$ to obtain an approximation for $0.998^5 + 1.005^2$

**[2 marks]**

**5** *ABC* is a triangle. The point *D* lies on *AC*.

$AB = 8$ cm, $BC = BD = 7$ cm and angle $A = 60°$ as shown in the diagram.




**5 (a)** Using the cosine rule, find the length of *AC*.

**[3 marks]**

**5 (b)** Hence, state the length of *AD*.

**[1 mark]**

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**6** Find the solution to

$$5^{(2x+4)} = 9$$

giving your answer in the form $a + \log_5 b$, where $a$ and $b$ are integers.

**[3 marks]**

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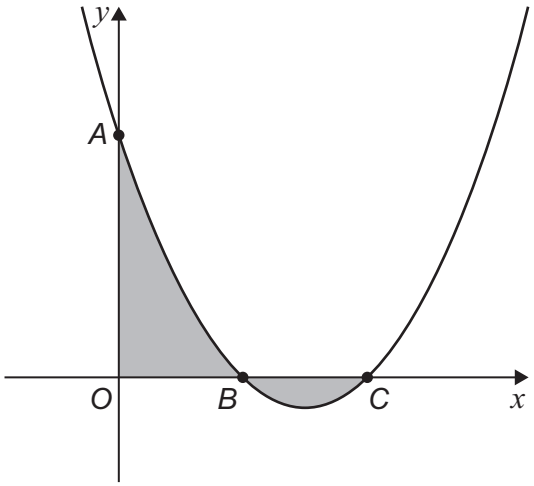
**7** The diagram below shows the graph of the curve that has equation $y = x^2 - 3x + 2$ along with two shaded regions.




**7 (a)** State the coordinates of the points *A*, *B* and *C*.

**[2 marks]**

**7 (b)** Katy is asked by her teacher to find the total area of the two shaded regions.

Katy uses her calculator to find $\int_0^2 (x^2 - 3x + 2)\,\mathrm{d}x$ and gets the answer $\frac{2}{3}$

Katy's teacher says that her answer is incorrect.

**7 (b) (i)** Show that the total area of the two shaded regions is 1

Fully justify your answer.

**[5 marks]**

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**7 (b) (ii)** Explain why Katy's method was not valid.

**[1 mark]**

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**8** It is given that $y = 3x - 5x^2$

Use differentiation from first principles to find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$

**[4 marks]**

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**9 (a)** Express $n^3 - n$ as a product of three factors.

**[1 mark]**

**9 (b)** Given that $n$ is a positive integer, prove that $n^3 - n$ is a multiple of 6

**[3 marks]**

**Turn over for the next question**

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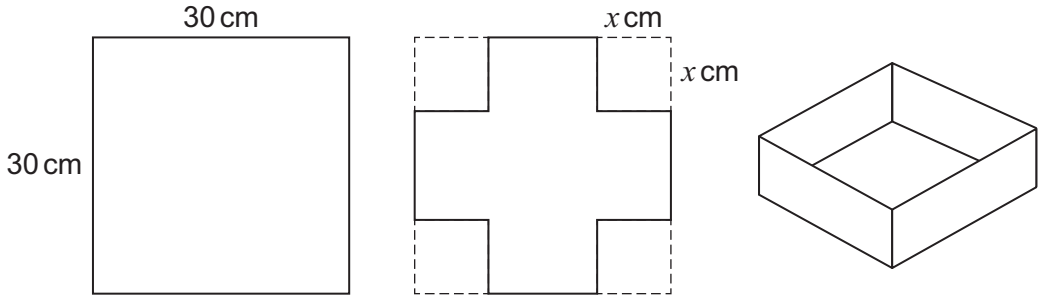
**10** A square sheet of metal has edges 30 cm long.

Four squares each with edge $x$ cm, where $x < 15$, are removed from the corners of the sheet.

The four rectangular sections are bent upwards to form an open-topped box, as shown in the diagrams.




**10 (a)** Show that the capacity, $C$ cm$^3$, of the box is given by

$$C = 900x - 120x^2 + 4x^3$$

**[2 marks]**

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**10 (b)** Find the maximum capacity of the box.

Fully justify your answer.

**[7 marks]**

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**11** A circle $C$ has centre (0, 10) and radius $\sqrt{20}$

A line $L$ has equation $y = mx$

**11 (a) (i)** Show that the $x$-coordinate of any point of intersection of $L$ and $C$ satisfies the equation

$$(1 + m^2)x^2 - 20mx + 80 = 0$$

**[3 marks]**

**11 (a) (ii)** Find the values of $m$ for which the equation in part **(a)(i)** has equal roots.

**[3 marks]**

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**11 (b)** Two lines are drawn from the origin which are tangents to $C$.

Find the coordinates of the points of contact between the tangents and $C$.

**[4 marks]**

**Turn over for the next question**

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## Section B

Answer **all** questions in the spaces provided.

**12** The table below shows the total monthly rainfall (in mm) in England and Wales in a sample of six years.

The sample of six years was taken from a data set covering every year from 1768 to 2018.

| | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1768** | 109.2 | 129.1 | 12.8 | 85.6 | 46.1 | 148.7 | 121.9 | 91.6 | 136.8 | 119.4 | 142.5 | 103.6 |
| **1818** | 98.0 | 65.8 | 134.7 | 135.6 | 55.9 | 31.2 | 50.4 | 21.0 | 115.6 | 75.8 | 112.0 | 46.8 |
| **1868** | 99.9 | 62.2 | 71.1 | 61.4 | 36.7 | 16.5 | 20.0 | 106.7 | 90.2 | 95.6 | 61.4 | 185.6 |
| **1918** | 91.2 | 61.6 | 36.7 | 63.3 | 58.5 | 30.9 | 110.0 | 62.9 | 189.5 | 69.1 | 66.3 | 122.5 |
| **1968** | 85.8 | 47.6 | 59.5 | 68.8 | 78.7 | 94.0 | 107.8 | 72.2 | 148.1 | 99.0 | 69.6 | 84.2 |
| **2018** | 104.5 | 52.8 | 115.1 | 91.4 | 51.9 | 16.5 | 39.6 | 76.7 | 67.0 | 75.8 | 104.9 | 116.0 |

Deduce the sampling method **most likely** to have been used to collect this sample.

Circle your answer.

**[1 mark]**

Opportunity  Simple Random  Stratified  Systematic

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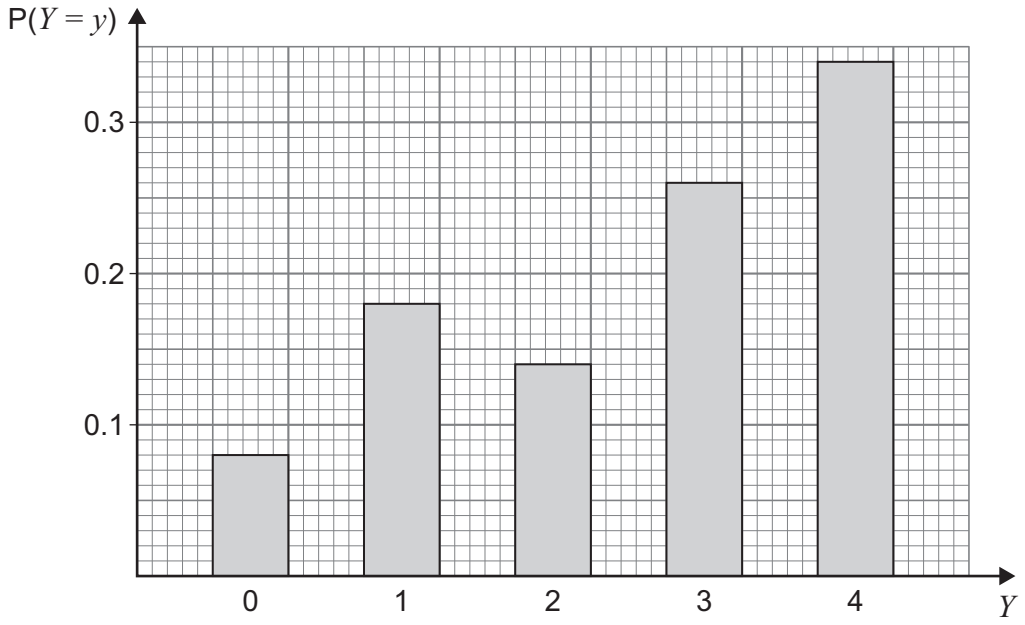
**13** The diagram below shows the probability distribution for a discrete random variable $Y$.




Find $P(0 < Y \leq 3)$.

Circle your answer.

**[1 mark]**

0.40 0.42 0.58 0.66

**Turn over for the next question**

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**14** The random variable $T$ follows a binomial distribution where

$$T \sim \text{B}(16, 0.3)$$

The mean of $T$ is denoted by $\mu$.

**14 (a)** Find $\text{P}(T \leq \mu)$.

**[2 marks]**

**14 (b)** Find the variance of $T$.

**[1 mark]**

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**Turn over for the next question**

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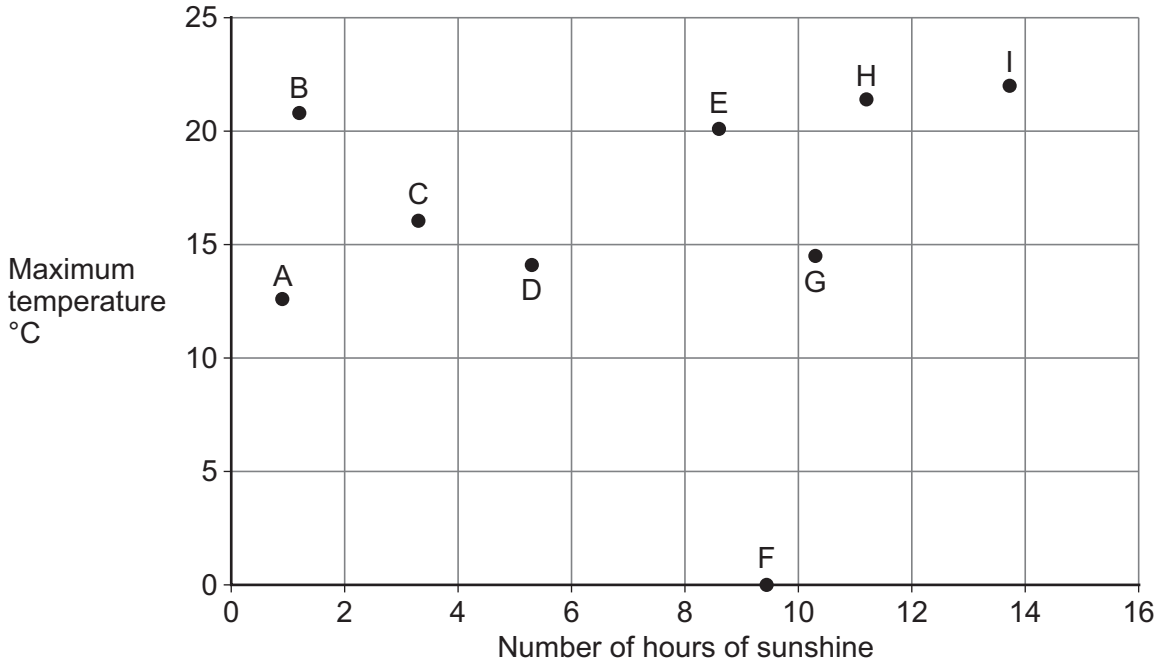
**15** The number of hours of sunshine and the daily maximum temperature were recorded over a 9-day period in June at an English seaside town.

A scatter diagram representing the recorded data is shown below.




One of the points on the scatter diagram is an error.

**15 (a) (i)** Write down the letter that identifies this point.

**[1 mark]**

_____

_____

_____

**15 (a) (ii)** Suggest one possible action that could be taken to deal with this error.

**[1 mark]**

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**15 (b)** It is claimed that the scatter diagram proves that longer hours of sunshine cause higher maximum daily temperatures.

Comment on the validity of this claim.

**[1 mark]**

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**Turn over for the next question**

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**16** An analysis was carried out using the Large Data Set to compare the $CO_2$ emissions (in g/km) from 2002 and 2016.

The summary statistics for the $CO_2$ emissions, $X$, for all cars registered as owned by either females or males is given in the table below.

| | **2002** | **2016** |
|---|---|---|
| $\sum x$ | 207 901 | 142 103 |
| **Sample size** | 1215 | 1144 |

**16 (a)** Find the reduction in the mean of the $CO_2$ emissions in 2016 compared to the mean of the $CO_2$ emissions in 2002.

**[2 marks]**

**16 (b)** It is claimed that the move to more electric and gas/petrol powered cars has caused the reduction in the mean $CO_2$ emissions found in part **(a)**.

Using your knowledge of the Large Data Set, state whether you agree with this claim.

Give a reason for your answer.

**[1 mark]**

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**16 (c)** There are 3827 data values in the Large Data Set.

It is claimed that the data in the table above must have been summarised incorrectly.

**16 (c) (i)** Explain why this claim is being made.

**[1 mark]**

_____

_____

_____

**16 (c) (ii)** State whether this claim is correct.

Give a reason for your answer.

**[1 mark]**

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**Turn over for the next question**

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**17** The number of toilets in each of a random sample of 200 properties from a town was recorded.

Four types of properties were included: terraced, semi-detached, detached and apartment.

The data is summarised in the table below.

| | **Number of toilets** | | |
|---|---|---|---|
| | **One** | **Two** | **Three** |
| **Terraced** | 20 | 10 | 4 |
| **Semi-Detached** | 18 | 50 | 16 |
| **Detached** | 12 | 10 | 8 |
| **Apartment** | 22 | 30 | 0 |

One of the properties is selected at random.

$A$ is the event 'the property has exactly two toilets'.

$B$ is the event 'the property is detached'.

**17 (a) (i)** Find P($A$).

**[1 mark]**

**17 (a) (ii)** Find P($A' \cap B$).

**[1 mark]**

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**17 (a) (iii)** Find $P(A \cup B)$.

**[2 marks]**

**17 (b)** Determine whether events $A$ and $B$ are independent.

Fully justify your answer.

**[2 marks]**

**17 (c)** Using the table, write down two events, **other than event $A$ and event $B$**, which are mutually exclusive.

**[1 mark]**

Event 1 _____

Event 2 _____

**Turn over for the next question**

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**18** It is known from previous data that 14% of the visitors to a particular cookery website are under 30 years of age.

To encourage more visitors under 30 years of age a large advertising campaign took place to target this age group.

To test whether the campaign was effective, a sample of 60 visitors to the website was selected. It was found that 15 of the visitors were under 30 years of age.

**18 (a)** Explain why a one-tailed hypothesis test should be used to decide whether the sample provides evidence that the campaign was effective.

**[1 mark]**

**18 (b)** Carry out the hypothesis test at the 5% level of significance to investigate whether the sample provides evidence that the proportion of visitors under 30 years of age has increased.

**[5 marks]**

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**18 (c)** State one necessary assumption about the sample for the distribution used in part **(b)** to be valid.

**[1 mark]**

_____

_____

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**END OF QUESTIONS**

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