AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ______________________________

Forename(s) ______________________________

Candidate signature ______________________________
I declare this is my own work.

A-level
MATHEMATICS
Paper 1

Tuesday 6 June 2023 Afternoon Time allowed: 2 hours

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
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TOTAL	

Answer **all** questions in the spaces provided.

1 Find the coefficient of x^7 in the expansion of $(2x-3)^7$

Circle your answer.

[1 mark]

-2187 -128 2 128

2 Given that $y = 2x^3$ find $\frac{dy}{dx}$

Circle your answer.

[1 mark]

$\frac{dy}{dx} = 5x^2$ $\frac{dy}{dx} = 6x^2$ $\frac{dy}{dx} = \frac{x^4}{2}$ $\frac{dy}{dx} = 6x^3$

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3 The curve with equation $y = \ln x$ is transformed by a stretch parallel to the x-axis with scale factor 2

Find the equation of the transformed curve.

Circle your answer.

[1 mark]

$y = \frac{1}{2}\ln x$ $\qquad$ $y = 2\ln x$ $\qquad$ $y = \ln\frac{x}{2}$ $\qquad$ $y = \ln 2x$

4 Given that θ is a small angle, find an approximation for $\cos 2\theta$

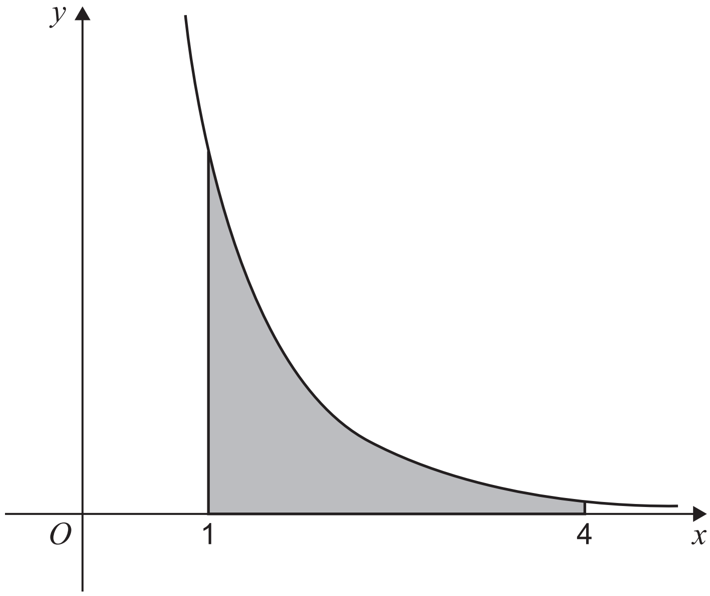
Circle your answer.

[1 mark]

$1 - \frac{\theta^2}{2}$ $\qquad$ $2 - 2\theta^2$ $\qquad$ $1 - 2\theta^2$ $\qquad$ $1 - \theta^2$

Turn over for the next question

5 The graph of $y = \dfrac{5}{e^x - 1}$ is shown in the diagram below.



The trapezium rule with 6 ordinates (5 strips) is to be used to find an approximation for the shaded area.

The values required to obtain this approximation are shown in the table below.

x	1	1.6	2.2	2.8	3.4	4
y	2.90988	1.26485	0.62305	0.32374	0.17263	0.09329

5 (a) Use the trapezium rule with 6 ordinates (5 strips) to find an approximate value for the shaded area.

Give your answer to four decimal places.

[3 marks]

5 (b) Using your answer to part **(a)** deduce an estimate for $\int_1^4 \frac{20}{e^x - 1}\,dx$

[1 mark]

Turn over for the next question

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6 Show that the equation

$$2\log_{10} x = \log_{10} 4 + \log_{10} (x+8)$$

has exactly one solution.

Fully justify your answer.

[5 marks]

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7 (a) Given that n is a positive integer, express

$$\frac{7}{3+5\sqrt{n}} - \frac{7}{5\sqrt{n}-3}$$

as a single fraction not involving surds.

[3 marks]

7 (b) Hence, deduce that

$$\frac{7}{3+5\sqrt{n}} - \frac{7}{5\sqrt{n}-3}$$

is a rational number for all positive integer values of n

[1 mark]

Do not write outside the box

8 Show that

$$\int_0^{\frac{\pi}{2}} (x \sin 4x)\, \mathrm{d}x = -\frac{\pi}{8}$$

[6 marks]

Turn over for the next question

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9 The points P and Q have coordinates $(-6, 15)$ and $(12, 19)$ respectively.

9 (a) (i) Find the coordinates of the midpoint of PQ

[1 mark]

9 (a) (ii) Find the equation of the perpendicular bisector of PQ

Give your answer in the form $ax + by = c$ where a, b and c are integers.

[4 marks]

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9 (b) (i) A circle passes through the points *P* and *Q*

The centre of the circle lies on the line with equation $2x - 5y = -30$

Find the equation of the circle.

[3 marks]

9 (b) (ii) The circle intersects the coordinate axes at n points.

State the value of n

[1 mark]

Turn over ▶

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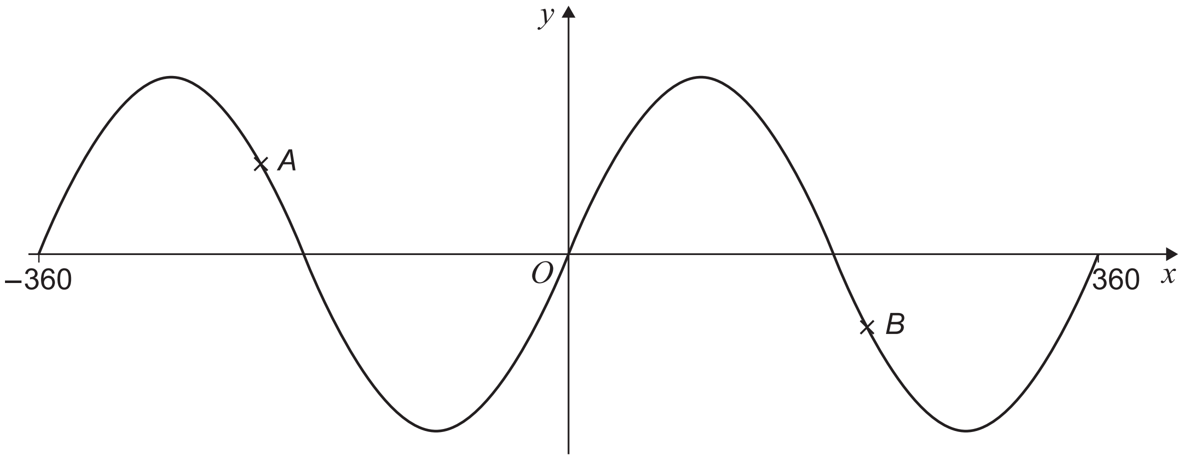
10 The curve with equation

$$y = \sin x°$$

for $-360 \leq x \leq 360$ is shown below.



10 (a) Point *A* on the curve has coordinates (a, 0.5)

10 (a) (i) Find the value of a

[2 marks]

10 (a) (ii) State the value of $\sin(180° - a°)$

[1 mark]

Do not write outside the box

10 (b) Point B on the curve has coordinates $\left(b,\ -\frac{3}{7}\right)$

10 (b) (i) Find the exact value of $\sin(b° - 180°)$

[2 marks]

10 (b) (ii) Find the exact value of $\cos b°$

[3 marks]

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11 The nth term of a sequence is u_n

The sequence is defined by

$$u_{n+1} = pu_n + 70$$

where $u_1 = 400$ and p is a constant.

11 (a) Find an expression, in terms of p, for u_2

[1 mark]

11 (b) It is given that $u_3 = 382$

11 (b) (i) Show that p satisfies the equation

$$200p^2 + 35p - 156 = 0$$

[3 marks]

11 (b) (ii) It is given that the sequence is a decreasing sequence.

Find the value of u_4 and the value of u_5

[3 marks]

11 (c) The limit of u_n as n tends to infinity is L

11 (c) (i) Write down an equation for L

[1 mark]

11 (c) (ii) Find the value of L

[1 mark]

12 One of the rides at a theme park is a room where the floor and ceiling both move up and down for 10π seconds.

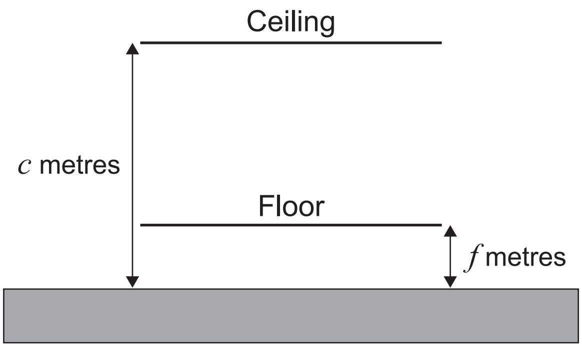
At time t seconds after the ride begins, the distance f metres of the floor above the ground is

$$f = 1 - \cos t$$

At time t seconds after the ride begins, the distance c metres of the ceiling above the ground is

$$c = 8 - 4 \sin t$$

The ride is shown in the diagram below.



12 (a) Show that the initial distance between the floor and ceiling is 8 metres.

[1 mark]

12 (b) Show that the distance d metres between the floor and ceiling at time t is given by

$$d = 7 + R\cos(t + \alpha)$$

where R and α are positive constants to be found.

[5 marks]

12 (c) Hence, find the minimum distance between the ceiling and the floor.

Give your answer to the nearest centimetre.

[2 marks]

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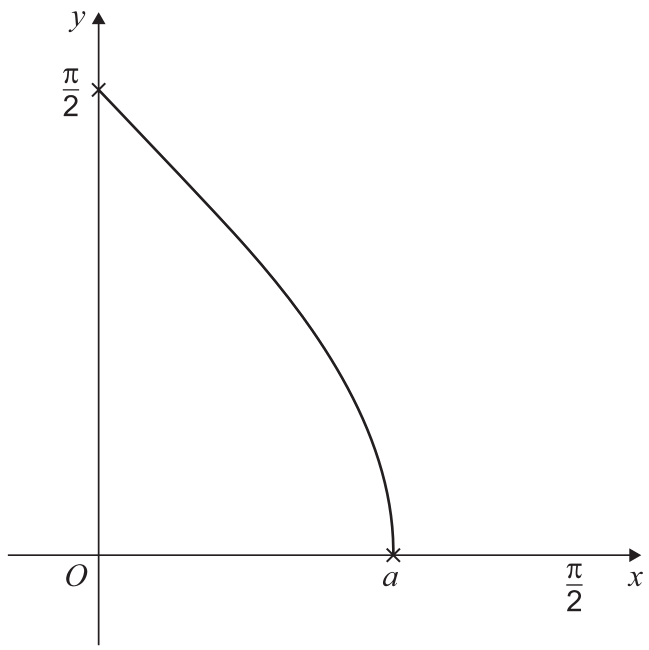
13 The function f is defined by

$$f(x) = \arccos x \quad \text{for} \quad 0 \leq x \leq a$$

The curve with equation $y = f(x)$ is shown below.



13 (a) State the value of a

[1 mark]

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13 (b) (i) On the diagram above, sketch the curve with equation

$$y = \cos x \quad \text{for} \quad 0 \leq x \leq \frac{\pi}{2}$$

and

sketch the line with equation

$$y = x \quad \text{for} \quad 0 \leq x \leq \frac{\pi}{2}$$

[4 marks]

13 (b) (ii) Explain why the solution to the equation

$$x - \cos x = 0$$

must also be a solution to the equation

$$\cos x = \arccos x$$

[1 mark]

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13 (c) Use the Newton-Raphson method with $x_0 = 0$ to find an approximate solution, x_3, to the equation

$$x - \cos x = 0$$

Give your answer to four decimal places.

[3 marks]

Turn over for the next question

Turn over ▶

Do not write outside the box

14 (a) (i) Given that

$$y = 2^x$$

write down $\frac{dy}{dx}$

[1 mark]

14 (a) (ii) Hence find

$$\int 2^x \, dx$$

[2 marks]

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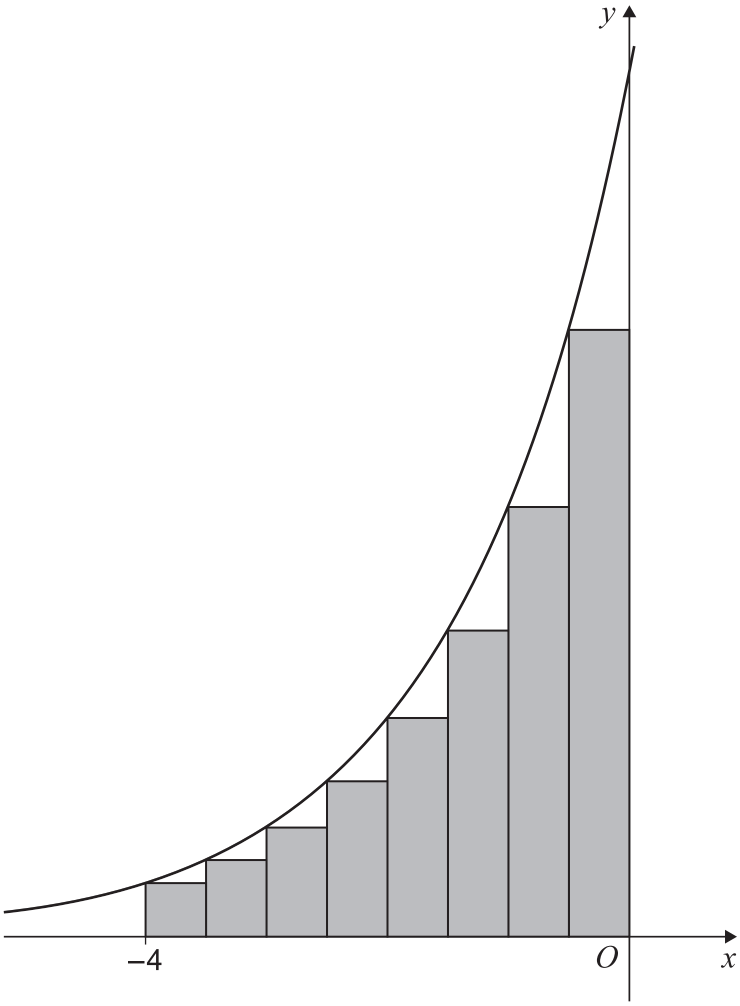
14 (b) The area, A, bounded by the curve with equation $y = 2^x$, the x-axis, the y-axis and the line $x = -4$ is approximated using eight rectangles of equal width as shown in the diagram below.



14 (b) (i) Show that the exact area of the largest rectangle is $\dfrac{\sqrt{2}}{4}$

[2 marks]

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Question 14 continues on the next page

14 (b) (ii) The areas of these rectangles form a geometric sequence with common ratio $\frac{\sqrt{2}}{2}$

Find the exact value of the total area of the eight rectangles.

Give your answer in the form $k(1+\sqrt{2})$ where k is a rational number.

[3 marks]

14 (b) (iii) More accurate approximations for A can be found by increasing the number, n, of rectangles used.

Find the exact value of the limit of the approximations for A as $n \to \infty$

[3 marks]

Turn over for the next question

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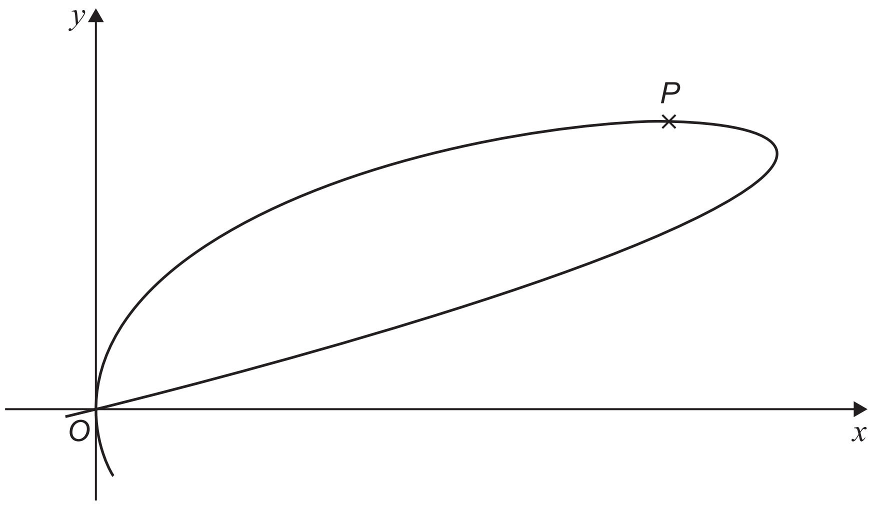
15 The curve with equation

$$x^2 + 2y^3 - 4xy = 0$$

has a single stationary point at *P* as shown in the diagram below.



15 (a) Show that the y-coordinate of *P* satisfies the equation

$$y^2(y - 2) = 0$$

[7 marks]

15 (b) Hence, find the coordinates of P

[2 marks]

Turn over for the next question

Do not write outside the box

16 (a) Given that

$$\frac{1}{16-9x^2} \equiv \frac{A}{4-3x} + \frac{B}{4+3x}$$

find the values of A and B

[3 marks]

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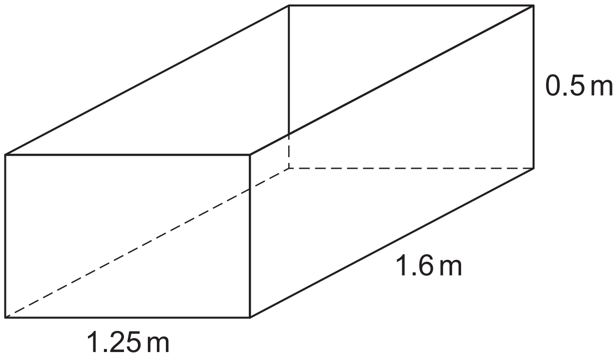
16 (b) An empty container, in the shape of a cuboid, has length 1.6 metres, width 1.25 metres and depth 0.5 metres, as shown in the diagram below.



The container has a small hole in the bottom.

Water is poured into the container at a rate of 0.16 cubic metres per minute.

At time t minutes after the container starts to be filled, the depth of water is d metres and water leaks out at a rate of $0.36d^2$ cubic metres per minute.

At time t minutes after the container starts to be filled, the volume of water in the container is V cubic metres.

16 (b) (i) Show that

$$\frac{dV}{dt} = \frac{16 - 9V^2}{100}$$

[4 marks]

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16 (b) (ii) Hence, find t in terms of V

[5 marks]

16 (b) (iii) Determine how long it takes to fill the container with water.

Give your answer to the nearest minute.

[2 marks]

END OF QUESTIONS

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