A-level
MATHEMATICS
7357/2
Paper 2

Mark scheme

June 2023

Version: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:
- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
‘their’	Indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)
ISW	Ignore Subsequent Working

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to students showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the student to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

Q	Marking instructions	AO	Marks	Typical solution
1	Ticks correct box	2.5	B1	$\{x : x < 2\} \cup \{x : x > 5\}$
	Question 1 Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
2	Circles correct answer	2.2a	R1	30
	Question 2 Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
3	Circles correct answer	1.1b	B1	4
	Question 3 Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
4(a)	Writes $\sqrt{x}$ as $x^{\frac{1}{2}}$ PI by derivative with $kx^{-\frac{1}{2}}$	1.1b	B1	$y=\frac{x^2}{8}+4x^{\frac{1}{2}}$ $\frac{dy}{dx}=\frac{x}{4}+2x^{-\frac{1}{2}}$
	Differentiates with at least one term correct	1.1a	M1	
	Obtains a correct expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ ACF ISW	1.1b	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
4(b)	Obtains gradient of 2 or Substitutes x = 4 into their expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$	1.1a	M1	$\frac{\mathrm{d}y}{\mathrm{d}x}=2$ $y-10=2(x-4)$
	Obtains correct equation of the tangent. Does not need to be fully simplified. ACF For example: $y=2x+2$ ISW	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
4(c)	Equates their $\frac{\mathrm{d}y}{\mathrm{d}x}$ to zero. OE	1.1a	M1	$\frac{x}{4}+2x^{-\frac{1}{2}}=0$ $\frac{x}{4}+\frac{2}{\sqrt{x}}=0$ As $x>0$, $\frac{2}{\sqrt{x}}>0$ and $\frac{x}{4}>0$ Therefore $\frac{x}{4}+\frac{2}{\sqrt{x}}>0$ Equation has no solutions so the curve has no stationary points.
	Completes reasoned argument by correctly manipulating the equation to obtain or $x^{\frac{3}{2}}=-8$ or $x^{\frac{1}{2}}=-2$ **and** states $x=4$ is a solution **and** then deduces $\frac{x}{4}+\frac{2}{\sqrt{x}}=2\neq 0$ or the gradient found at $x=4$ in part (b) was non-zero **and** concludes that the curve has no stationary points. or Completes reasoned argument by correctly manipulating the equation to obtain $x^2=-8\sqrt{x}$ or $x^{\frac{3}{2}}=-8$ or $x^{\frac{1}{2}}=-2$ **and** deduces the equation has no solutions by making explicit reference to $\sqrt{x}>0$ **and** concludes that the curve has no stationary points. or Completes reasoned argument to establish that $\frac{x}{4}+\frac{2}{\sqrt{x}}>0$ **and** deduces the equation has no solutions **and** concludes that the curve has no stationary points.	2.1	R1	
	Subtotal		2	

	Question 4 Total		7	

Q	Marking instructions	AO	Marks	Typical solution
5(a)	Forms a correct expression for the number of lengths swum on the nth day. ACF Can be unsimplified. For example: $10+4(n-1)$	1.1b	B1	$4n+6$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
5(b)(i)	Forms the linear equation $25\times$ their part(a) expression $=3000$ OE Condone incorrect inequalities	3.1b	M1	$4n+6=\dfrac{3000}{25}$ $n=28.5$ Ziad will need to train for 29 days
	Solves their linear equation and rounds or truncates to the nearest positive integer. Condone incorrect inequalities	3.2a	M1	
	Obtains 29 CAO	1.1b	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
5(b)(ii)	Uses a **correct** formula for the sum to n terms of an arithmetic progression substituting a = 10 **and** d = 4 or l = 122	3.4	M1	$\dfrac{29}{2}\left(2\times10+\left(29-1\right)\times4\right)\times25=47850$ Swims 47850 metres 47850 < 50 000 Therefore the coach is not correct
	Obtains either 47850 metres or 1914 lengths or AWRT 29.7 days. OE Condone missing units	3.2a	A1	
	Makes an appropriate comparison and concludes that the coach is wrong. The comparison must be explicit and can be one of the following: 47850 < 50 000 1914 < 2000 29.7 > 29 or 30 > 29 FT $\boldsymbol{n}$ **= 28 only** with one of the comparisons 44800 < 50 000 1792 < 2000 29.7 > 28 or 30 > 28 This latter case would be a maximum of M1 A0 R1F	2.4	R1F	
	Subtotal		**3**	

	Question 5 Total		**7**	

Q	Marking instructions	AO	Marks	Typical solution
6(a)(i)	Writes down at least one of the following: $\log_{10} a = 1.76$ or $\log a = 1.76$ or $a = 10^{1.76}$ to show that a is AWRT 57.5 AG	1.1b	B1	$\log_{10} a = 1.76$ $a = 10^{1.76}$ $a = 57.5$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
6(a)(ii)	Obtains b = 1.14 AWRT 1.14	1.1b	B1	b = 1.14
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
6(b)	Obtains their $100(b-1)$ FT their b where b > 1	3.2a	B1F	14%
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
6(c)(i)	Substitutes N = 16 into $\log_{10} V = 0.057N + 1.76$ or Substitutes N = 16 into $V = a \times b^N$ using their b value and $a = 57.5$ or AWRT 57.5 PI AWRT 467.9 or 469.9	3.4	M1	$V = 57.5 \times 1.14^{16}$ $= 467.9$ £467 900 000
	Obtains a value in the interval [£467 800 000, £470 000 000] Must include £ or pounds. Accept use of millions. For example: £467.9 million.	3.2a	A1	
	Subtotal		**2**	

<table>
<tr><th>Q</th><th>Marking instructions</th><th>AO</th><th>Marks</th><th>Typical solution</th></tr>
<tr><td>6(c)(ii)</td><td>Gives a reason, in context, why extrapolation from the model may not be valid.

Must include reference to sales or shopping.
For example:
Sales increased in 2020 due to the pandemic.
Sales would be impacted by supply shortages.
More people shopping in person after a pandemic.
People shop in supermarkets instead of online as technology becomes too expensive.
Accept any specific reference to an event since 2016 that would impact on sales/shopping.</td><td>3.5b</td><td>E1</td><td>Sales may suddenly fall due to unforeseen circumstances such as a pandemic.</td></tr>
<tr><td></td><td>Subtotal</td><td></td><td>1</td><td></td></tr>
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	Question 6 Total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
7(a)	Obtains $\frac{1}{\sqrt{10-2x}}$ ACF	1.1b	B1	$\mathrm{h}(x)=\frac{1}{\sqrt{10-2x}}$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
7(b)	Deduces $x<5$ ACF Condone incorrect set notation	2.2a	B1	$x<5$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
7(c)	Forms the equation y = their $\mathrm{h}(x)$ and squares both sides of the equation to remove the square root correctly. or Forms the equation y = their $\mathrm{h}(x)$ and rearranges to obtain an expression for $\sqrt{10-2x}$ x and y can be switched at any point.	3.1a	M1	$y=\frac{1}{\sqrt{10-2x}}$ $\sqrt{10-2x}=\frac{1}{y}$ $10-2x=\frac{1}{y^2}$ $2x=10-\frac{1}{y^2}$ $x=5-\frac{1}{2y^2}$ $\mathrm{h}^{-1}(x)=5-\frac{1}{2x^2}$
	Obtains $10-2x=\frac{1}{y^2}$ or $5-x=\frac{1}{2y^2}$ x and y can be switched at any point.	1.1b	A1	
	Completes reasoned argument with no incorrect steps to show the given result. Must use correct notation $\mathrm{h}^{-1}(x)$ and be consistent with use of variables. AG	2.1	R1	
	Subtotal		**3**	

	Question 7 Total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
8(a)	Recalls $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$ PI by use of $\operatorname{cosec}^2\theta = \frac{1}{\sin^2\theta}$	1.2	B1	$\frac{1}{1-\cos\theta}+\frac{1}{1+\cos\theta}$ $\equiv \frac{1+\cos\theta+1-\cos\theta}{(1-\cos\theta)(1+\cos\theta)}$ $\equiv \frac{2}{1-\cos^2\theta}$ $\equiv \frac{2}{\sin^2\theta}$ $\equiv 2\operatorname{cosec}^2\theta$
	Recalls $\cos^2\theta+\sin^2\theta=1$ OE	1.2	B1	
	Forms a single fraction with a denominator of $(1-\cos\theta)(1+\cos\theta)$ OE	1.1a	M1	
	Completes reasoned argument using $\cos^2\theta+\sin^2\theta=1$ to prove the given identity. AG	2.1	R1	
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
8(b)	Forms the equation $2\operatorname{cosec}^2\theta = A$ or $\frac{2}{\sin^2\theta}=A$ OE PI by $A \geq 2$	1.1a	M1	$2\operatorname{cosec}^2\theta = A$ $\operatorname{cosec}\theta \leq -1$ or $\operatorname{cosec}\theta \geq 1$ Hence $\operatorname{cosec}^2\theta \geq 1$ $\therefore A \geq 2$
	Explains that $\operatorname{cosec}\theta \leq -1$, $\operatorname{cosec}\theta \geq 1$ or $\operatorname{cosec}^2\theta \geq 1$ or Explains that $-1 \leq \sin\theta \leq 1$ or $\sin^2\theta \leq 1$ Accept an accurate sketch of $y=\operatorname{cosec}^2\theta$ with 1 labelled on the y-axis Condone strict inequalities.	2.4	E1	
	Deduces $A \geq 2$	2.2a	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
8(c)	Uses the identity from part (a) to obtain $2(1+\cot^2\theta)=16$ or $\operatorname{cosec}^2\theta=8$ or $\sin^2\theta=\frac{1}{8}$ or $\cos^2\theta=\frac{7}{8}$	1.1a	M1	$2\operatorname{cosec}^2\theta=16$ $\operatorname{cosec}^2\theta=8$ $1+\cot^2\theta=8$ $\cot^2\theta=7$ $\cot\theta=-\sqrt{7}$ since θ is obtuse
	Obtains $\cot^2\theta=7$ PI by $\cot\theta=\sqrt{7}$ or $\cot\theta=-\sqrt{7}$	1.1b	A1	
	Deduces $\cot\theta=-\sqrt{7}$	2.2a	R1	
	Subtotal		**3**	

Q	Marking instructions (Modified Question Paper only)	AO	Marks	Typical solution
8(c)	Uses the identity from part (a) to obtain $\sin^2\theta=\frac{1}{8}$ or Rearranges $\frac{1}{1-\cos\theta}+\frac{1}{1+\cos\theta}=16$ to obtain $\cos^2\theta=\frac{7}{8}$	1.1a	M1	$2\operatorname{cosec}^2\theta=16$ $\operatorname{cosec}^2\theta=8$ $\sin^2\theta=\frac{1}{8}$ $1-\cos^2\theta=\frac{1}{8}$ $\cos^2\theta=\frac{7}{8}$ $\cos\theta=-\sqrt{\frac{7}{8}}$ since θ is obtuse
	Obtains $\cos\theta=\sqrt{\frac{7}{8}}$ or $\cos\theta=-\sqrt{\frac{7}{8}}$ OE Must be exact.	1.1b	A1	
	Deduces $\cos\theta=-\sqrt{\frac{7}{8}}$ OE Must be exact.	2.2a	R1	
	Subtotal		**3**	

	Question 8 Total		**10**	

Q	Marking instructions	AO	Marks	Typical solution
9(a)	Uses the binomial expansion to obtain either $\left(-\frac{1}{2}\right)x$ or $\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)x^2}{2!}$ OE	1.1a	M1	$(1+x)^{-\frac{1}{2}} \approx 1+\left(-\frac{1}{2}\right)x+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)x^2}{2!}$ $\approx 1-\frac{1}{2}x+\frac{3}{8}x^2$
	Obtains $1-\frac{1}{2}x+\frac{3}{8}x^2$ Must have evaluated coefficients – allow equivalent fractions.	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
9(b)	Explains that the expansion is only valid for $\lvert x\rvert<1$ OE Accept that the expansion is not valid for $\lvert x\rvert>1$ Must include the word **valid** or **invalid**.	2.3	E1	The expansion is valid for $\lvert x\rvert<1$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
9(c)	Substitutes $x=-\frac{1}{4}$ into their answer to part (a)	1.1a	M1	$1-\frac{1}{2}\left(-\frac{1}{4}\right)+\frac{3}{8}\left(-\frac{1}{4}\right)^2=\frac{147}{128}$ $\left(\frac{3}{4}\right)^{-\frac{1}{2}}=\frac{2}{\sqrt{3}}$ $\frac{2}{\sqrt{3}}=\frac{147}{128}$ $\frac{1}{\sqrt{3}}=\frac{147}{256}\approx 0.574$
	Obtains $\frac{147}{128}$ AWRT 1.148 Condone 1.15 if a fully correct substituted expansion is seen.	1.1b	A1	
	Deduces the value 0.574 AWRT 0.574 or Deduces the value 0.580 AWRT 0.580	2.2a	A1	
	Subtotal		**3**	

	Question 9 Total		**6**	

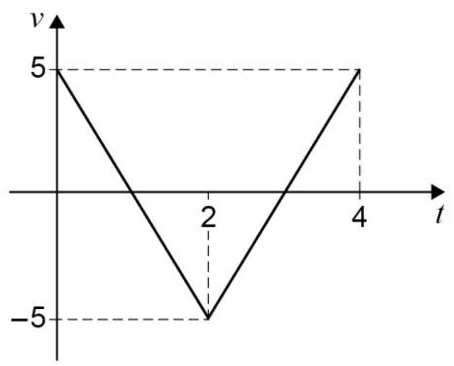
Q	Marking instructions	AO	Marks	Typical solution
10(a)	Obtains $a^2 - 2ab + b^2$	1.1b	B1	$a^2 - 2ab + b^2$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
10(b)	Forms a different sum of a non-zero rational and its reciprocal.	1.1a	M1	$-2 + \frac{1}{-2} = -\frac{5}{2}$ $-\frac{5}{2} < 2$
	Finds a correct counter example **and** compares the result with 2 Must have used 1 or a negative value.	2.3	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
10(c)	Forms the inequality $\frac{a}{b} + \frac{b}{a} \le 2$ (for a pair of distinct positive integers a and b) Condone $\frac{a}{b} + \frac{b}{a} < 2$	2.1	M1	Assume $\frac{a}{b} + \frac{b}{a} \le 2$ $\frac{a^2 + b^2}{ab} \le 2$ $a^2 + b^2 \le 2ab$ $a^2 - 2ab + b^2 \le 0$ $(a-b)^2 \le 0$ Since $a \ne b$ this is a contradiction because $(a-b)^2 > 0$ Hence $\frac{a}{b} + \frac{b}{a} > 2$
	Rearranges and factorises to deduce $(a-b)^2 \le 0$ Condone $(a-b)^2 < 0$	2.2a	A1	
	Completes a reasoned argument to explain the contradiction. Must have started with $\frac{a}{b} + \frac{b}{a} \le 2$ and stated $a \ne b$ or makes reference to them being distinct.	2.1	R1	
	Subtotal		**3**	

	Question 10 Total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
11	Circles correct answer	1.2	B1	$0.2g$ N
	Question 11 Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
12	Ticks correct box	2.2a	B1	
	Question 12 Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
13(a)	Selects an appropriate equation of constant acceleration to find t and uses $u = u$ and $v = 3u$	1.1a	M1	$v = u + at$
	Completes reasoned argument show the given result. **Must** have clearly stated $u = u \quad v = 3u \quad a = g$ and **must** see either $3u = u + gt$ or $\frac{3u - u}{g} = t$ AG	2.1	R1	$u = u \qquad v = 3u \qquad a = g$ $3u = u + gt$ $\frac{3u - u}{g} = t$ $t = \frac{2u}{g}$
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
13(b)	Selects a correct equation of constant acceleration to find s and substitutes correctly. Condone $a = -g$	3.3	M1	$v^2 = u^2 + 2as$ $u = u \qquad v = 3u \qquad a = g$ $(3u)^2 = u^2 + 2gs$
	Completes reasoned argument with at least one more intermediate step to obtain $\frac{4u^2}{g}$	1.1b	A1	$9u^2 = u^2 + 2gs$ $8u^2 = 2gs$
	Explains that have found the distance MN **and** N is not on the surface to justify $h > \frac{4u^2}{g}$ AG	2.4	R1	$MN = s = \frac{4u^2}{g}$ Since N is above the ground then $h > \frac{4u^2}{g}$
	Subtotal		**3**	

	Question 13 Total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
14	Integrates a with at least one term correct.	3.4	M1	$v=\int a\,\mathrm{d}t$
	Obtains a fully correct expression for v ACF Coefficients can be unsimplified. Condone omission of constant	1.1b	A1	$v=kt^3-kt^2+t+c$ $v=1$ when $t=0$ then $c=1$ v = 10 and t = 3
	Uses given initial conditions to find their constant of integration. This must be done **before** v = 10 and t = 3 are substituted.	3.4	M1	$10=27k-9k+3+1$ $18k=6$ $k=\frac{1}{3}$
	Completes reasoned argument by substituting v = 10 and t = 3 into $v=kt^3-kt^2+t+1$ to show $k=\frac{1}{3}$ Must include at least more one intermediate step after substituting. AG	1.1b	A1	
	Question 14 Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
15	Uses $m=\frac{W}{g}$ PI by sight of AWRT 0.07	1.1b	B1	$F=\mu R=0.4\times 0.65=0.26$
	States or uses $F=\mu R$ PI by sight of 0.26	1.1a	M1	$m=\frac{0.65}{9.8}=0.066$
	Forms a three term equation using $a=0.91$, their F and their m	3.3	M1	$D-F=ma$ $D-0.26=0.066\times 0.91$
	Forms fully correct equation with all values substituted correctly to obtain D = 0.32 AWRT 0.32	1.1b	A1	$D=0.32$
	Question 15 Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
16	Adds the two forces together. ACF	1.1b	B1	$\mathbf{F}_1 + \mathbf{F}_2 = \begin{bmatrix} 1.6+k \\ 5k-5 \end{bmatrix}$ $\begin{bmatrix} 1.6+k \\ 5k-5 \end{bmatrix} = m\begin{bmatrix} 3.2 \\ 12 \end{bmatrix}$ $1.6+k = 3.2m$ $5k-5 = 12m$ $k = 8.8$
	Uses $\mathbf{F} = m\boldsymbol{a}$ and substitutes their $\mathbf{F}_1 \pm \mathbf{F}_2$ and $\boldsymbol{a} = \begin{bmatrix} 3.2 \\ 12 \end{bmatrix}$ PI by use of ratios For example: $\dfrac{5k-5}{12} = \dfrac{1.6+k}{3.2}$ OE	3.1a	M1	
	Obtains two correct equations For example: $1.6+k = 3.2m$ **and** $5k-5 = 12m$ Only award if vectors are removed or Obtains a correct linear equation in k For example: $\dfrac{5k-5}{12} = \dfrac{1.6+k}{3.2}$ OE PI by $k = 8.8$ or $m = 3.25$	1.1b	A1	
	Obtains $k = 8.8$ ACF	1.1b	A1	
	Question 16 Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
17(a)(i)	Forms a moments equation by taking moments about Y with one term correct. or Forms two **correct** moments equations by taking moments about P and Q Must clearly use force $\times$distance for every term. For example: Moments about P $3.5mg = (1.4)(4g) + 5R$ Moments about Q $3.5mg = (5.6)(4g) + 2R$ Moments about the centre $1.5(Yg) = (2.1)(4g)$	3.1b	M1	Taking moments about Y $1.5mg = (3.6)(4g)$ $m = \frac{(3.6)(4g)}{1.5g}$ $m = 9.6$ kilograms
	Completes reasoned argument with at least one intermediate step to obtain 9.6 kilograms AG Condone 9.6	2.1	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
17(a)(ii)	Resolves vertically to find R or Forms a moments equation about any point other than about Y with the correct number of terms and at least one term correct. For example: Moments about X $(2.1)(9.6g) = 3.6R$ Moments about midpoint of plank $(2.1)(4g) = 1.5R$ PI by $5.6g$	3.3	M1	Resolving vertical forces: $4g + R = 9.6g$ $R = 5.6g$ N
	Obtains $5.6g$ N Condone missing units.	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
17(b)	Obtains 4.8g or States that the two supports are equidistant from the centre/the ends of the plank or Refers to symmetry	3.1b	B1	X and Y are the same distance from the midpoint, so their reaction forces are equal. The reaction force at Y decreases. Therefore, the claim is wrong.
	States one of the following: • The **reaction force at Y** changes. • The **reaction force at Y** decreases. • $4.8g \neq 5.6g$ • $4.8g < 5.6g$ **and** concludes that the claim is incorrect.	2.4	E1	
	Subtotal		**2**	

	Question 17 Total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
18(a)	Obtains correct speed of $\sqrt{12}$ OE AWRT 3.46 Condone missing units	1.1b	B1	Speed = $\sqrt{(3)^2 + (\sqrt{3})^2}$ = $2\sqrt{3}$ m s^{-1}
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
18(b)	Uses $\tan^{-1}\dfrac{\sqrt{3}}{3}$ or $\tan^{-1}\dfrac{3}{\sqrt{3}}$ to find the angle between one of the velocity vectors relative to the **i** direction or the **j** direction. Sight of sine rule or cosine rule using a magnitude for AC scores M0 R0	3.1a	M1	
	Completes a reasoned argument to obtain 30^0 for both angles relative to the **i** direction and adds them together to obtain angle ABC = 60^0 or Completes a reasoned argument to obtain 60^0 for both angles relative to the **j** direction and adds them together and subtracts them from 180^0 to obtain angle ABC = 60^0 Solution must include clear reference to angle ABC or indicate angle ABC with a letter on a diagram. When using trigonometric ratios the vectors **i** and **j** must not be included.	2.1	R1	Angle between AB and **i** direction $= \tan^{-1}\dfrac{\sqrt{3}}{3} = 30^0$ Angle between BC and **i** direction $= \tan^{-1}\dfrac{\sqrt{3}}{3} = 30^0$ Angle ABC = $30^0 + 30^0 = 60^0$
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
18(c)	Deduces time taken from B to C is 3 seconds.	2.2a	R1	Time taken from B to C = $\frac{9}{3} = 3$
	Obtains an expression for displacement from B to C of the form $t\begin{bmatrix}-3\\\sqrt{3}\end{bmatrix}$ **where** $1 < t \leq 9$	3.1a	M1	$\overline{OC} = \begin{bmatrix}1\\0\end{bmatrix} + 3\begin{bmatrix}-3\\\sqrt{3}\end{bmatrix}$
	Obtains $\begin{bmatrix}-8\\3\sqrt{3}\end{bmatrix}$	1.1b	A1	$= \begin{bmatrix}-8\\3\sqrt{3}\end{bmatrix}$
	Subtotal		**3**	

	Question 18 Total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
19(a)	Resolves the 2 N force to obtain either $2\cos 40$ AWRT 1.53 or $2\sin 40$ AWRT 1.29 May be seen on the diagram.	1.1a	M1	Use $F = ma$ for system $2\cos 40-(0.8+R)=2.2(0.06)$ $1.53-0.8-R=0.132$ $R\approx 0.6\text{ N}$
	Uses Newton's 2nd Law to form a four term equation for the whole system. This may be seen with total resistance equivalent to $0.8+R$ or Uses Newton's 2nd Law to form a three term equation for the trailer or a four term equation for the engine. Condone one incorrect sign.	3.3	M1	
	Obtains a fully correct equation for the whole system. or Obtains two fully correct equations for the train engine and the trailer. For example: $2\cos 40-T-0.8=0.09$ $T-R=0.042$	3.3	A1	
	Completes a reasoned argument to show that the given value for R is approximately 0.6 AG	2.1	R1	
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
19(b)(i)	Forms an equation of motion **without** a driving force for the engine or the combined system. PI by $a=\frac{7}{11}$ or $-\frac{7}{11}$	3.3	M1	$-0.8-T=1.5a$ $T-0.6=0.7a$ $T=\frac{17}{110}$ N
	Obtains one of the following: $\pm(T-0.6)=0.7a$ or $\mp(0.8+T)=1.5a$ or $\pm(0.8+0.6)=2.2a$ Allow use of $a=-\frac{7}{11}$	3.4	M1	
	Obtains a correct pair of equations of motion for the trailer and the engine which are both moving in the same direction. or Obtains $a=-\frac{7}{11}$ and one fully correct equation of motion involving T	1.1b	A1	
	Finds T AWFW [0.15 , 0.16]	1.1b	A1	
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
19(b)(ii)	Obtains $a=-\frac{7}{11}$ AWRT -0.64 Condone missing or incorrect units	3.1b	B1	$a=-\frac{7}{11}$ $0=0.5^2+2ah$ $h=\frac{11}{56}\approx 0.20$
	Selects an appropriate equation of constant acceleration to find s and substitutes u = 0.5 , v = 0 and their a Do not accept a = - g	1.1a	M1	
	Obtains required distance. AWRT 0.2 ISW	1.1b	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
19(c)	States one appropriate modelling assumption about the rod Accept Rod is rigid OE	3.5b	E1	The rod is horizontal.
	Subtotal		**1**	

	Question 19 Total		**12**	

Q	Marking instructions	AO	Marks	Typical solution
20	States or uses $14\cos 60$ for the horizontal component.	3.1b	B1	$u_H = 14\cos 60 = 7$
	States or uses $14\sin 60$ for the vertical component.	3.1b	B1	$u_V = 14\sin 60 = 7\sqrt{3}$
	Uses $s = ut + \frac{1}{2}at^2$ with u = their vertical component of velocity, a = -g and $s = \pm 1.5$ OE PI by t = AWFW [2.54, 2.60]	3.3	M1	$s = 7\sqrt{3}\,t - \frac{g}{2}t^2$ $-1.5 = 7\sqrt{3}\,t - 4.9t^2$
	Obtains t = 2.592 AWFW [2.54, 2.60] Exact value is $\frac{3\sqrt{10}+5\sqrt{3}}{7}$	1.1b	A1	$4.9t^2 - 12.124t - 1.5 = 0$ $t = 2.592$ seconds $7t = 18.144$
	Multiplies their t value by their horizontal component provided their t > 0.2	1.1b	M1	$ut + \frac{1}{2}at^2 = 0.5a(2.392)^2$
	Substitutes u = 0, their t – 0.2 into $ut + \frac{1}{2}at^2$ to obtain an expression for the horizontal distance travelled by the dog.	3.3	M1	$2.860832a = 18.144$ $a = 6.3$
	Obtains 6.3 CAO	3.2a	A1	
	Question 20 Total		**7**	

	Question Paper Total		**100**	