A-level

MATHEMATICS

7357/1

Paper 1

Mark scheme

June 2023

Version: Final 1.1

236A7357/1/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:
- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	Indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)
ISW	Ignore Subsequent Working

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to students showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the student to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses . If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

Q	Marking instructions	AO	Marks	Typical solution
1	Circles the correct answer	1.1b	B1	128
	Question 1 Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
2	Circles the correct answer	1.1b	B1	$\frac{\mathrm{d}y}{\mathrm{d}x} = 6x^2$
	Question 2 Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
3	Circles the correct answer	2.2a	R1	$y = \ln\frac{x}{2}$
	Question 3 Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
4	Circles the correct answer	2.2a	R1	$1-2\theta^2$
	Question 4 Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
5(a)	States or uses h = 0.6 OE Accept 0.3 OE as the multiplier. PI by correct answer	1.1b	B1	$\frac{0.6}{2}\begin{pmatrix}2.90988+0.09329+\\ 2(1.26485+0.62305+0.32374+0.17263)\end{pmatrix}$ $=2.3315$
	Substitutes given y values to achieve $2.90988+0.09329+$ $2(1.26485+0.62305+0.32374+0.17263)$ Ignore h. Accept correct exact values or values to more than 5 decimal places. PI by correct answer or 7.77171	1.1a	M1	
	Obtains 2.3315 AWRT	1.1b	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
5(b)	Obtains 4x their answer to (a) correct to at least 2 significant figures.	2.2a	R1F	9.3
	Subtotal		**1**	

	Question 5 Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
6	Uses power log rule correctly Or Raises 10 to the power of both sides and correctly obtains $10^{\log_{10} x^2}$ or x^2 PI by correct quadratic	1.1b	B1	$2\log_{10} x = \log_{10} 4 + \log_{10}(x+8)$ $\log_{10} x^2 = \log_{10} 4(x+8)$ $x^2 = 4x + 32$ $x^2 - 4x - 32 = 0$ $x = -4$ or 8 -4 is not a solution as $\log_{10} -4$ has no real value. Therefore, the equation has exactly one solution.
	Uses addition or subtraction log rule correctly. Or Correctly combines two indices. PI by correct quadratic.	1.1b	B1	
	Solves a three-term quadratic equation obtaining at least one real value for x	1.1a	M1	
	Obtains $x = 8$ Must have scored B1,B1,M1.	1.1b	A1	
	Obtains correct values of x and explains why -4 is not a solution. Must refer to the log of a negative or state it is only possible to find the log of a positive. Accept correct reference to the domain of a log function. Must have achieved B1,B1,M1,A1	2.4	E1	
	Question 6 Total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
7(a)	Multiplies both the numerator and denominator of at least one of the given fractions by an appropriate conjugate. Or Obtains a common denominator with numerators which simplify to $35\sqrt{n}-21-\left(21+35\sqrt{n}\right)$ or a single fraction with numerator $35\sqrt{n}-21-\left(21+35\sqrt{n}\right)$	1.1a	M1	$\frac{7}{3+5\sqrt{n}}-\frac{7}{5\sqrt{n}-3}$ $=\frac{7\left(5\sqrt{n}-3\right)}{\left(3+5\sqrt{n}\right)\left(5\sqrt{n}-3\right)}-\frac{7\left(3+5\sqrt{n}\right)}{\left(3+5\sqrt{n}\right)\left(5\sqrt{n}-3\right)}$ $=\frac{35\sqrt{n}-21-\left(21+35\sqrt{n}\right)}{\left(3+5\sqrt{n}\right)\left(5\sqrt{n}-3\right)}$ $=-\frac{42}{25n-9}$
	Obtains a correct single unsimplified fraction.	1.1b	A1	
	Obtains correct simplified fraction $-\frac{42}{25n-9}$ OE	2.1	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
7(b)	Explains that the numerator and denominator are both integers, or rational, and concludes it is rational	2.4	E1F	Since 42 and $25n$-9 are both integers the expression is rational
	Subtotal		**1**	

	Question 7 Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
8	Begins integration by parts by writing $u = x \quad v' = \sin 4x$ $u' = 1 \quad v = A\cos 4x$ PI by $Ax\cos 4x - A\int (\cos 4x)\,\mathrm{d}x$ Or $u = \sin 4x \quad v' = x$ $u' = A\cos 4x \quad v = Bx^2$ PI by $Px^2 \sin 4x - Q\int (x^2\cos 4x)\,\mathrm{d}x$	3.1a	M1	$u = x \quad v' = \sin 4x$ $u' = 1 \quad v = -\frac{1}{4}\cos 4x$ $\left[-\frac{1}{4}x\cos 4x\right]_0^{\frac{\pi}{2}} + \frac{1}{4}\int_0^{\frac{\pi}{2}} (\cos 4x)\,\mathrm{d}x$ $= \left[-\frac{1}{4}x\cos 4x + \frac{1}{16}\sin 4x\right]_0^{\frac{\pi}{2}}$ $= \left(-\frac{\pi}{8}\cos\frac{4\pi}{2} + \frac{1}{16}\sin\frac{4\pi}{2}\right) - \left(0 \times \cos 0 + \frac{1}{16}\sin 0\right)$ $= -\frac{\pi}{8}$
	Selects the correct method for integration by parts $u = x \quad v' = \sin 4x$ $u' = 1 \quad v = A\cos 4x$ PI by $Ax\cos 4x - A\int (\cos 4x)\,\mathrm{d}x$	1.1a	M1	
	Substitutes their u, u', v, v' of **either** of the above forms into the integration by parts formula. Eg $Px\cos 4x - P\int (\cos 4x)\,\mathrm{d}x$ $Px^2 \sin 4x - Q\int (x^2\cos 4x)\,\mathrm{d}x$ $\frac{x^2}{2}\sin 4x - \int (2x^2\cos 4x)\,\mathrm{d}x$ PI by $-\frac{1}{4}x\cos 4x + \frac{1}{16}\sin 4x$	1.1a	M1	
	Obtains $-\frac{1}{4}x\cos 4x - \frac{1}{4}\int (-\cos 4x)\,\mathrm{d}x$ Condone missing dx PI by $-\frac{1}{4}x\cos 4x + \frac{1}{16}\sin 4x$	1.1b	A1	
	Completes integration by parts to obtain $-\frac{1}{4}x\cos 4x + B\sin 4x$ with $B \neq \pm 1$	1.1a	M1	
	Completes reasoned argument by explicitly substituting correct limits into $-\frac{1}{4}x\cos 4x + \frac{1}{16}\sin 4x$	2.1	R1	

	To obtain $-\frac{\pi}{8}$ Accept $\left(-\frac{\pi}{8}\cos 2\pi+\frac{1}{16}\sin 2\pi\right)-0$ AG			
	Question 8 Total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
9(a)(i)	Obtains $(3,17)$ Condone position vectors, missing brackets or $x = 3$ and $y = 17$	1.1b	B1	$(3,17)$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
9(a)(ii)	Obtains gradient of PQ PI correct gradient used in equation of perpendicular bisector.	1.1b	B1	$m_{PQ} = \dfrac{19-15}{12--6} = \dfrac{4}{18}$ $y-17 = -\dfrac{9}{2}(x-3)$ $2y-34 = -9x+27$ $9x+2y = 61$
	Forms an equation of a line Either using the negative reciprocal of their gradient **or** their midpoint	3.1a	M1	
	Forms an equation of a line using the negative reciprocal of their gradient **and** their midpoint	1.1a	M1	
	Obtains $9x+2y=61$ OE in the required form.	2.1	A1	
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
9(b)(i)	Solves simultaneously using their $9x+2y=61$ from (a)(ii) with $2x-5y=-30$ to obtain the centre of the circle PI by $(5,8)$ or $x = 5$, $y = 8$	3.1a	M1	Centre $(5,8)$ $(x-5)^2+(y-8)^2 = r^2$ $(12-5)^2+(19-8)^2 = 170$ $(x-5)^2+(y-8)^2 = 170$
	Uses *P* or *Q* and their centre to find the radius or radius^2	3.1a	M1	
	Obtains $(x-5)^2+(y-8)^2=170$ ACF Eg $x^2-10x+y^2-16y=81$	1.1b	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
9(b)(ii)	States 4 Must have the correct centre and correct radius or radius^2	2.2a	R1	4
	Subtotal		**1**	

	Question 9 Total		**9**	

Q	Marking instructions	AO	Marks	Typical solution
10(a)(i)	States or uses sin 30 = 0.5 PI by sight of ± 150 or ± 30 or -330 Maybe seen on diagram	1.1a	M1	–180-30 = -210
	Obtains -210	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
10(a)(ii)	Obtains 0.5	1.1b	B1	0.5
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
10(b)(i)	Uses a correct approach to find $\sin(b-180)$. Might see $\sin(205.37...\pm 180)$ PI by correct answer or $\sin(\pm 180 - 25.376...)$ PI by correct answer or Correct use of compound angle formula PI by correct answer	3.1a	M1	$\sin(b-180) = -\sin b$ $= \frac{3}{7}$
	Deduces $\sin(b-180) = \frac{3}{7}$ CAO	2.2a	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
10(b)(ii)	Uses $\cos^2 x + \sin^2 x = 1$ or Draws right angled triangle with 3 and 7 on opp and hyp sides. PI by $\cos b = -\frac{2\sqrt{10}}{7}$ OE exact form	3.1a	M1	$\cos^2 b + \left(-\frac{3}{7}\right)^2 = 1$ $\cos^2 b = \frac{40}{49}$ $\cos b = -\frac{2\sqrt{10}}{7}$
	Obtains $\cos^2 b = \frac{40}{49}$ Condone b replaced by different variable or obtains a ratio for cosine of the correct exact magnitude.	1.1b	A1	
	Deduces $\cos b = -\frac{2\sqrt{10}}{7}$ OE exact form CAO	2.2a	R1	
	Subtotal		**3**	

	Question 10 Total		**8**	

Q	Marking instructions	AO	Marks	Typical solution
11(a)	Obtains $400p + 70$	1.1b	B1	$400p + 70$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
11(b)(i)	Substitutes 382 **or** their u_2 into $u_3 = pu_2 + 70$	1.1a	M1	$382 = pu_2 + 70$ $382 = p(400p + 70) + 70$ $382 = 400p^2 + 70p + 70$ $400p^2 + 70p - 312 = 0$ $200p^2 + 35p - 156 = 0$
	Substitutes 382 **and** their u_2 into $u_3 = pu_2 + 70$ To obtain a quadratic equation in p PI by $382 = p(400p + 70) + 70$	3.1a	M1	
	Obtains correct equation and rearranges to obtain given answer. Must see brackets expanded before given answer.	2.1	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
11(b)(ii)	Obtains both p = 0.8 and -0.975 PI by correct $u_4 = 375.6$ **and** $u_5 = 370.48$	1.1b	B1	$p = 0.8,\ \ p = -0.975$ $p = -0.975$ $\Rightarrow u_4 = -302.45,\ u_5 = 364.88875$ not decreasing $p = 0.8$ $\Rightarrow u_4 = 375.6,\ u_5 = 370.48$
	Uses p = 0.8 or -0.975 to obtain a value for u_4 PI by 375.6, -302.45, 370.48 Accept equivalent fractions or AWRT 364.89	3.1a	M1	
	Deduces correct values for u_4 and u_5. $(u_4 =)375.6$ and $(u_5 =)370.48$ Accept equivalent fractions If incorrect values are seen they must be rejected.	2.2a	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
11(c)(i)	Forms the equation $L = pL + 70$ or $(1-p)L = 70$ Or with $p =$ 0.8 or -0.975 substituted into either of these equations accept if $1-p$ is evaluated ISW	3.1a	B1	$L = 0.8L + 70$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
11(c)(ii)	Deduces the value of L is 350 or AWRT 35.4 Accept $\frac{2800}{79}$ or both	2.2a	R1	350
	Subtotal		**1**	

	Question 11 Total		**9**	

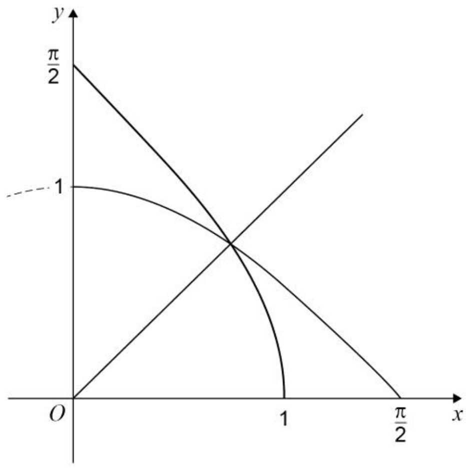
Q	Marking instructions	AO	Marks	Typical solution
12(a)	Substitutes t = 0 into both models and obtains the distance. Condone missing units.	3.4	B1	$t=0$ $8-4\sin 0-(1-\cos 0)=8$ metres
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
12(b)	Models the distance between the ceiling and the floor as $c-f$ Condone a sign error when expressions for c and f are substituted.	3.3	M1	$d=c-f$ $=8-4\sin t-(1-\cos t)$ $=7+\cos t-4\sin t$ $R=\sqrt{17}$ $R\cos\alpha=1$ $R\sin\alpha=4$ $\tan\alpha=4, \alpha=1.33$ $d=7+\sqrt{17}\cos(t+1.33)$
	Uses a compound angle formula to obtain $R\cos\alpha=\pm1 \, or \pm 4$ or $R\sin\alpha=\pm4 \, or \pm1$ or $\tan\alpha=\pm4 \, or \pm\frac{1}{4}$ PI by $R=\sqrt{17}\approx4.1$ or AWRT $\alpha=1.33^c$ or AWRT $\alpha=76^\circ$	3.1a	M1	
	Obtains $R=\sqrt{17}\approx4.1$ Condone correct answer from $\sqrt{(\pm1)^2+(\pm4)^2}$ Note: M0 M1 A1 is possible	1.1b	A1	
	Obtains AWRT $\alpha=1.33^c$ or AWRT $\alpha=76^\circ$ No incorrect working seen in finding α Accept other valid values of α $\alpha=1.33^c+2n\pi$ OE in degrees	1.1b	A1	
	Completes argument to obtain $d=7+\sqrt{17}\cos(t+1.33)$ Accept AWRT 4.1 in place of $\sqrt{17}$ and $\alpha=1.33^c+2n\pi$ **Do not** award this mark if $\sin\alpha=\pm4 \, or \pm1$ or $\cos\alpha=\pm1 \, or \pm4$ is used leading to a value of $\tan\alpha$	2.1	R1	
	Subtotal		**5**	

Q	Marking instructions	AO	Marks	Typical solution
12(c)	Subtracts their R from 7 provided their R < 7	1.1a	M1	$7-\sqrt{17}=2.88\,\text{m}$
	Obtains 2.88 metres or 288 cm Correct units must be seen.	3.2a	A1	
	Subtotal		**2**	

	Question 12 Total		**8**	

Q	Marking instructions	AO	Marks	Typical solution
13(a)	States 1	1.2	B1	1
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
13(b)(i)	Draws a concave arc for $0 \le x \le \frac{\pi}{2}$ Must intersect y-axis below $\frac{\pi}{2}$ Condone dotted section	1.1a	M1	
	Labels the y-intercept of their concave arc 1 or a.	1.1b	A1	
	Draws straight line through O at approximately $45°$ crossing the given curve $y = \arccos x$	1.1b	M1	
	Shows all three graphs intersecting at a common point with the maximum of the cosine graph in the correct position and $y = x$ shown as a straight line through O.	2.2a	A1	
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
13(b)(ii)	Explains that $y = \cos x$ and $y = \arccos x$ are reflections in $y = x$ Accept $y = x$ is a line of symmetry. Accept all **three** graphs meet at the same point. Or Starts with $x = \cos x$ and obtains $\arccos x = x$ Accept $\cos^{-1} x$ for $\arccos x$ throughout.	2.4	E1	All three graphs intersect at the same point.
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
13(c)	Obtains $1+\sin x$ PI by $x_2 = 0.75036...$ AWRT 0.75	1.1b	B1	$x_{n+1} = x_n - \dfrac{x_n - \cos x_n}{1+\sin x_n}$ $x_3 = 0.7391$
	Obtains $x_n - \dfrac{x_n - \cos x_n}{1 \pm \sin x_n}$ Ignore subscripts, condone ANS for x_n PI by $x_2 = 0.75036...$ AWRT 0.75	1.1a	M1	
	Obtains AWRT $x_3 = 0.7391$ condone missing label provided this is their final answer. Must have scored M1.	1.1b	A1	
	Subtotal		**3**	

	Question 13 Total		**9**	

Q	Marking instructions	AO	Marks	Typical solution
14(a)(i)	Obtains $2^x \ln 2$ Or $\ln 2 e^{x\ln 2}$	1.2	B1	$\frac{dy}{dx} = 2^x \ln 2$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
14(a)(ii)	Integrates to obtain $k2^x,\ k \neq 1 or 0$ OE	1.1a	M1	$\int 2^x dx = \frac{2^x}{\ln 2} + c$
	Deduces $\int 2^x dx = \frac{2^x}{\ln 2} + c$ OE Must include +c	2.2a	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
14(b)(i)	Obtains $2^{-\frac{1}{2}}$ Exact value ACF	1.1a	M1	$\frac{1}{2} \times \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{4}$
	Writes the product $0.5 \times 2^{-\frac{1}{2}}$ in exact form ACF to obtain given answer. Condone $-0.5 \times 2^{-\frac{1}{2}}$ if reason given for rejecting the negative sign	2.1	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
14(b)(ii)	Uses $S_n = \dfrac{a\left(1-r^n\right)}{1-r}$ With at least two of $a = \dfrac{\sqrt{2}}{4}$, $r = \dfrac{\sqrt{2}}{2}$ and n = 8 correct Or with at least two of $a = \dfrac{1}{32}$, $r = \sqrt{2}$ and n = 8 correct Or Forms the sum of 8 rectangles using $\dfrac{1}{2}\left(\begin{array}{l}\dfrac{1}{\sqrt{2}}+\dfrac{1}{2}+\dfrac{1}{2\sqrt{2}}+\dfrac{1}{4}+\\ \dfrac{1}{4\sqrt{2}}+\dfrac{1}{8}+\dfrac{1}{8\sqrt{2}}+\dfrac{1}{16}\end{array}\right)$ OE with at least 4 correct terms	1.1a	M1	$\dfrac{\dfrac{\sqrt{2}}{4}\left(1-\left(\dfrac{\sqrt{2}}{2}\right)^8\right)}{1-\dfrac{\sqrt{2}}{2}}$ $= \dfrac{15+15\sqrt{2}}{32}$ $= \dfrac{15}{32}\left(1+\sqrt{2}\right)$
	Obtains a correct expression can be left unsimplified.	1.1b	A1	
	Obtains $\dfrac{15\left(1+\sqrt{2}\right)}{32}$ or $\dfrac{15}{32}\left(1+\sqrt{2}\right)$ Do not award value of k is just stated without either of these answers.	2.1	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
14(b)(iii)	Forms the definite integral $\int_{-4}^{0} 2^x \mathrm{d}x$ PI by $\frac{1}{\ln 2}\left[2^x\right]_{-4}^{0}$ Condone swapped limits and missing dx PI by AWRT ± 1.35	3.1a	M1	$\int_{-4}^{0} 2^x \mathrm{d}x = \frac{1}{\ln 2}\left[2^x\right]_{-4}^{0}$ $= \frac{1}{\ln 2}\left(2^0 - 2^{-4}\right)$ $= \frac{15}{16\ln 2}$
	Substitutes 0 and -4 correctly into the correct integrated expression Or Obtains AWRT 1.35	1.1b	A1	
	Obtains correct exact value ACF	1.1b	A1	
	Subtotal		**3**	

	Question 14 Total		**11**	

Q	Marking instructions	AO	Marks	Typical solution
15(a)	Differentiates x^2 to obtain $2x$	1.1b	B1	
	Uses implicit differentiation and obtains either $Ay^2\frac{\mathrm{d}y}{\mathrm{d}x}$ or $Bx\frac{\mathrm{d}y}{\mathrm{d}x}$ terms	3.1a	M1	
	Uses product rule to obtain $\pm 4y \pm 4x\frac{\mathrm{d}y}{\mathrm{d}x}$ condone sign errors	1.1a	M1	
	Obtains $2x+6y^2\frac{\mathrm{d}y}{\mathrm{d}x}-4y-4x\frac{\mathrm{d}y}{\mathrm{d}x}=0$ OE	1.1b	A1	$x^2+2y^3-4xy=0$ $2x+6y^2\frac{\mathrm{d}y}{\mathrm{d}x}-4y-4x\frac{\mathrm{d}y}{\mathrm{d}x}=0$
	Substitutes $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ into their differentiated expression which contains either $Ay^2\frac{\mathrm{d}y}{\mathrm{d}x}$ or $Bx\frac{\mathrm{d}y}{\mathrm{d}x}$ PI by later work.	1.1a	M1	At stationary point $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ So $2x-4y=0$
	Deduces $x=2y$ or $2x=4y$ or $-x=-2y$ or $-2x=-4y$ Must have scored A1 with no incorrect rearrangement of $2x+6y^2\frac{\mathrm{d}y}{\mathrm{d}x}-4y-4x\frac{\mathrm{d}y}{\mathrm{d}x}=0$ used.	2.2a	R1	$x=2y$ $(2y)^2+2y^3-4(2y)y=0$ $4y^2+2y^3-8y^2=0$ $2y^3-4y^2=0$ $y^2(y-2)=0$
	Eliminates x in given equation and completes reasoned argument with at least one intermediate step to show the given result. Must have scored first 6 marks with no incorrect rearrangement of $2x+6y^2\frac{\mathrm{d}y}{\mathrm{d}x}-4y-4x\frac{\mathrm{d}y}{\mathrm{d}x}=0$ used.	2.1	R1	
	Subtotal		**7**	

Q	Marking instructions	AO	Marks	Typical solution
15(b)	Obtains y-coordinate of 2 Accept $y = 2$	1.1b	B1	(4, 2)
	Obtains x-coordinate of 4 Accept $x = 4$	1.1b	B1	
	Subtotal		**2**	

	Question 15 Total		**9**	

Q	Marking instructions	AO	Marks	Typical solution
16(a)	Uses a suitable method and finds a value for A or B. For example Rearranges and substitutes values or compares coefficients Or Uses cover-up method Or Uses inspection PI by A correct or B correct.	1.1a	M1	$\frac{1}{(4-3x)(4+3x)} \equiv \frac{A}{4-3x} + \frac{B}{4+3x}$ $1 \equiv A(4+3x) + B(4-3x)$ Let $x = -\frac{4}{3}$ $1 \equiv 8B \Rightarrow B = \frac{1}{8}$ comparing x terms $A = B = \frac{1}{8}$
	Obtains $A = \frac{1}{8}$	1.1b	A1	
	Obtains $B = \frac{1}{8}$	1.1b	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
16(b)(i)	Forms differential equation using $\frac{\mathrm{d}V}{\mathrm{d}t} = \pm 0.16 \pm 0.36d^2$	3.3	M1	$\frac{\mathrm{d}V}{\mathrm{d}t} = 0.16 - 0.36d^2$ $V = 1.25 \times 1.6d \Rightarrow d = \frac{V}{2}$ $\frac{\mathrm{d}V}{\mathrm{d}t} = 0.16 - 0.36\left(\frac{V}{2}\right)^2$ $= 0.16 - 0.09V^2$ $= \frac{16-9V^2}{100}$
	Obtains $V = 1.25 \times 1.6d$ OE	3.1b	B1	
	Substitutes their expression for d into $\frac{\mathrm{d}V}{\mathrm{d}t} = \pm 0.16 \pm 0.36d^2$ to obtain a differential equation in V and t only.	3.1a	M1	
	Completes reasoned argument to show the given result. AG	2.1	R1	
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
16(b)(ii)	Rearranges to obtain one of the following: $\dfrac{P}{16-9V^2}\mathrm{d}V=\dfrac{1}{Q}\mathrm{d}t$ $\dfrac{P}{16-9V^2}\dfrac{\mathrm{d}V}{\mathrm{d}t}=\dfrac{1}{Q}$ $\dfrac{P}{16-9V^2}=\dfrac{1}{Q}\dfrac{\mathrm{d}t}{\mathrm{d}V}$ where $P\times Q=100$ If their P = 100 no need to see $\dfrac{1}{Q}$ explicit with dt May include integral signs PI $\int\dfrac{100}{16-9V^2}\mathrm{d}V=t$	3.1a	B1	$\int\dfrac{1}{16-9V^2}\mathrm{d}V=\int\dfrac{1}{100}\mathrm{d}t$ $\dfrac{1}{8}\int\dfrac{1}{4-3V}+\dfrac{1}{4+3V}\mathrm{d}V=\int\dfrac{1}{100}\mathrm{d}t$ $\dfrac{1}{24}\left(-\ln\left(4-3V\right)+\ln\left(4+3V\right)\right)=\dfrac{t}{100}+c$ $t=0,V=0\Rightarrow c=0$ $\dfrac{100}{24}\left(-\ln\left(4-3V\right)+\ln\left(4+3V\right)\right)=t$
	Integrates their constant integrand correctly with respect to t. Follow through any constant.	1.1b	B1F	
	Writes $\int\dfrac{1}{16-9V^2}\mathrm{d}V$ as $\int\dfrac{A}{4-3V}+\dfrac{B}{4+3V}\mathrm{d}V$ Condone missing $\mathrm{d}V$ PI by $-\dfrac{A}{3}\ln\left(4-3V\right)+\dfrac{B}{3}\ln\left(4+3V\right)$	3.1a	M1	
	Integrates their partial fractions correctly to obtain $-\dfrac{A}{3}\ln\left(4-3V\right)+\dfrac{B}{3}\ln\left(4+3V\right)\quad(+c)$ OE Their A and B may be correctly inside the natural logs for example $\dfrac{1}{24}\left(-\ln\left(32-24V\right)+\ln\left(32+24V\right)\right)$	1.1b	A1F	
	Completes argument, including demonstrating that the constant of integration is zero.	2.1	R1	
	Subtotal		**5**	

Q	Marking instructions	AO	Marks	Typical solution
16(b)(iii)	Obtains a value for t by substituting $V=1$ into their expression for t from their final answer from b(ii) PI by 8 minutes from a correct answer from b(ii)	3.4	M1	$t=\frac{100}{24}\left(-\ln\left(4-3\right)+\ln\left(4+3\right)\right)$ $=\frac{100}{24}\ln 7$ $=8$ minutes
	Obtains 8 minutes following a correct answer in b(ii) Condone missing units	3.2a	A1	
	Subtotal		**2**	

	Question 16 Total		**14**	

	Question Paper Total		**100**	